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# Comparing GDP in Constant and in Chained Prices: Some New Results

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# Comparing GDP in constant and in chained prices: Some new results

**Jesus C. Dumagan**[*]

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## Abstract

This paper's framework for GDP in chained prices yields GDP in constant prices as a special case of constant relative prices, i.e., these GDP measures differ only when relative prices change. The framework has a novel additive procedure, counter to the prevailing view that GDP in chained prices is non-additive. This procedure allows relative prices to change but when they are constant, components in chained and in constant prices are equal, implying consistency with the additivity of GDP in constant prices. Finally, GDP conversion from constant to chained prices removes the fixed base─by making the immediately preceding period the base, i.e., continuous updating─and allows relative prices to change and, thus, removes the base-period dependence and substitution bias of GDP in constant prices.


*Keywords*: Real GDP, chained prices, constant prices, additivity, Fisher index
*JEL classification*: C43

## 1. Introduction

GDP in constant and in chained prices are alternative measures of *real* GDP. GDP in constant prices is real in the "physical" sense that it measures the economy's overall quantities *unadjusted* for relative price changes because it uses only the prices in the fixed base period. In contrast, GDP in chained prices is real in the "economic" sense that it measures overall quantities *adjusted* for relative price changes recognizing that the latter have real (e.g., substitution) effects.

From the above distinction, this paper's comparative analysis─between GDP in constant and in chained prices─proceeds as follows. Section 2 lays out an additive framework for GDP in chained prices and shows how to compute this GDP from normally available data on GDP components in current and in constant prices. This framework has a novel additive procedure, counter to the prevailing view that GDP in chained prices is non-additive (Ehemann, Katz, and Moulton, 2002; Whelan, 2002; Balk and Reich, 2008).[1]

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[1] In this paper, additivity means that levels of GDP components add up exactly to the level of overall GDP and that components' growth rate contributions also add up exactly to the overall growth rate. GDP in

Section 3 shows that GDP in constant prices is a special case of GDP in chained prices when relative prices are constant. It also highlights the problems of base-period dependence and substitution bias of GDP in constant prices that will, however, be removed by conversion to chained prices. Section 4 illustrates the analytic results with a numerical example. Section 5 concludes this paper with a summary of findings.

## 2. An additive framework for GDP in chained prices

This paper modifies the existing framework by the Bureau of Economic Analysis (BEA) for US GDP in chained prices (dollars) to achieve additivity fully because BEA's framework is additive in growth rates but not in levels and real shares.[2]

BEA employs the "superlative" Fisher (1922) quantity index ($Q_{st}^F$) denoted by the superscript $F$, using prices and quantities in the adjoining periods $s$ and $t$, i.e., $t = s + 1$.[3] $Q_{st}^F$ is the geometric mean of the Laspeyres quantity index ($Q_{st}^L$) denoted by the superscript $L$, and the Paasche quantity index ($Q_{st}^P$) denoted by the superscript $P$,

$$Q_{st}^F = (Q_{st}^L\, Q_{st}^P)^{\frac{1}{2}} \quad ; \quad Q_{st}^L = \frac{\sum_i^N p_{is} q_{it}}{\sum_i^N p_{is} q_{is}} \quad ; \quad Q_{st}^P = \frac{\sum_i^N p_{it} q_{it}}{\sum_i^N p_{it} q_{is}} \,. \tag{1}$$

To satisfy the convention that the index value equals 1 or 100 in the base period, an index $J_t$ is linked to the Fisher quantity index $Q_{st}^F$ such that,

$$J_t = J_s\, Q_{st}^F \quad ; \quad J_b = 1, \ \ b = \text{base period}\,. \tag{2}$$

Multiplying $J_t$ by GDP in the base period $b$ yields US GDP in chained prices in period $t$ given by $Y_t^F$ (Landefeld and Parker, 1997; Seskin and Parker, 1998; Moulton and Seskin, 1999),

$$Y_t^F = J_t \sum_i^N p_{ib} q_{ib} = J_t\, Y_b \ \ ; \ \ Y_s^F = J_s\, Y_b \ \ ; \ \ Y_b = \sum_i^N p_{ib} q_{ib} = \text{base-period GDP}. \tag{3}$$

It is important to note in (1) to (3) that the base period is not fixed for the indexes $J_s$, $J_t$, and $Q_{st}^F$ but continuously updated by making period $s$—immediately preceding the current period $t$—as the base. The base period $b$ serves only to denominate the *level* of GDP (i.e., in

constant prices is additive in both levels and growth rates. In contrast, in current practice, GDP in chained prices is additive in growth rates but not in levels, for example, in the case of US GDP. However, in section 2, this paper modifies the US GDP framework in chained prices to achieve additivity also in levels.

[2] BEA is the agency in the US Department of Commerce officially in charge of the compilation of US GDP. About the earliest appearance of US GDP in chained prices (dollars) may be found in BEA's official publication, *Survey of Current Business*, November/December 1995, p. 9.

[3] Diewert (1976) defined an index as "superlative" if it is *exact* for an aggregator function (e.g., a utility or production function) that is *flexible*, i.e., capable of providing a second-order differential approximation to an arbitrary twice differentiable linearly homogeneous function. The Fisher index is the exact index for the homogeneous form of the flexible quadratic aggregator function.

period $b$ prices) but $b$ has nothing to do with the GDP growth rate, growth contributions, and the real shares of components in chained prices. These are shown below.

It is shown first that GDP in chained prices is additive, starting with its growth rate from (2) and (3),

$$\frac{Y_t^F}{Y_s^F} - 1 = Q_{st}^F - 1\,. \tag{4}$$

The additive decomposition of the growth rate in (4) depends on the additive decomposition of the Fisher index ($Q_{st}^F$), which exists (van IJzeren, 1952; Dumagan, 2002; Balk, 2004). Dumagan (2002) showed that the additive decomposition of this index is,[4]

$$Q_{st}^F = \sum_i^N w_{is}^F \left(\frac{q_{it}}{q_{is}}\right) \quad ; \quad w_{is}^F = \left(\frac{P_{st}^F}{P_{st}^L + P_{st}^F}\right) w_{is}^L + \left(\frac{P_{st}^L}{P_{st}^L + P_{st}^F}\right) w_{is}^P \;; \tag{5}$$

$$w_{is}^L = \frac{p_{is}q_{is}}{\sum_i p_{is}q_{is}} \quad ; \quad w_{is}^P = \frac{p_{it}q_{is}}{\sum_i p_{it}q_{is}} \quad ; \quad \sum_i^N w_{is}^F = \sum_i^N w_{is}^L = \sum_i^N w_{is}^P = 1 \tag{6}$$

where $w_{is}^F$, $w_{is}^L$, and $w_{is}^P$ are the Fisher, Laspeyres, and Paasche weights of a component. Moreover, the Fisher price index ($P_{st}^F$) is the geometric mean of the Laspeyres price ($P_{st}^L$) and Paasche price ($P_{st}^P$) indexes,

$$P_{st}^F = (P_{st}^L\, P_{st}^P)^{\frac{1}{2}} \quad ; \quad P_{st}^L = \frac{\sum_i^N q_{is}p_{it}}{\sum_i^N q_{is}p_{is}} \quad ; \quad P_{st}^P = \frac{\sum_i^N q_{it}p_{it}}{\sum_i^N q_{it}p_{is}}\,. \tag{7}$$

It follows from (4) to (7) that the growth rate of GDP in chained prices can be decomposed by,

$$\frac{Y_t^F}{Y_s^F} - 1 = Q_{st}^F - 1 = \sum_i^N w_{is}^F \left(\frac{q_{it}}{q_{is}}\right) - 1 = \sum_i^N g_{it}^F \quad ; \quad g_{it}^F = w_{is}^F \left(\frac{q_{it}}{q_{is}} - 1\right) \tag{8}$$

from which $g_{it}^F$ is a component's additive growth contribution. Dumagan (2000) showed that $g_{it}^F$ is mathematically equivalent to BEA's formula (Moulton and Seskin, 1999) for a component's contribution to GDP growth in chained prices.

It follows from (8) that the level of GDP in chained prices can be decomposed by,

$$Y_t^F = Y_s^F \sum_i^N w_{is}^F \left(\frac{q_{it}}{q_{is}}\right) = \sum_i^N y_{it}^F \quad ; \quad y_{it}^F = Y_s^F w_{is}^F \left(\frac{q_{it}}{q_{is}}\right) \tag{9}$$

where $y_{it}^F$ is the additive level contribution of a component. For consistency with the additive growth contributions in (8), this paper proposes (9) for additive level contributions, in place of BEA's non-additive procedure for level contributions to US GDP in chained prices.

As noted above, BEA's growth contribution formula is equivalent to (8). However,

[4] Balk (2004) pointed out that van IJzeren (1952) was the first to derive a satisfactory *additive* decomposition, "unfortunately in an article in a rather obscure publication series of what is now called Statistics Netherlands." As a result, van IJzeren's additive decomposition escaped wider attention in the statistical community and, thus, Balk noted that "Dumagan (2002) independently rediscovered" this decomposition.

while (9) necessarily follows from (8), BEA does not use (9). In its place, BEA *replicates* (1), (2), and (3) to compute separate GDP subaggregates (e.g., consumption, investment, net exports, and government expenditures). The result is non-additivity because the Fisher index is not consistent in aggregation (Diewert, 1978). That is, BEA's separate GDP subaggregates in chained prices do not add up to the GDP in chained prices computed with *all* components included at once (Ehemann, Katz, and Moulton, 2002; Whelan, 2002).[5]

Finally, a component's real share is the ratio of $y_{it}^F$ to $Y_t^F$ in (9) which is,

$$\frac{y_{it}^F}{Y_t^F} = \frac{Y_s^F}{Y_t^F} w_{is}^F \left(\frac{q_{it}}{q_{is}}\right) = \frac{w_{is}^F}{Q_{st}^F}\left(\frac{q_{it}}{q_{is}}\right) \quad ; \quad \sum_i^N \frac{y_{it}^F}{Y_t^F} = 1\,. \tag{10}$$

Clearly, this paper's real shares are additive or sum to 1. In contrast, BEA's real shares will not sum to 1 because the subaggregates are not additive.

Thus, it appears that non-additivity or additivity is a *procedural* issue. Non-additivity is not inherent in the Fisher index because this index yields additive formulas for growth contributions in (8), level contributions in (9), and real shares in (10). In contrast, additivity is impossible in BEA's replication of the Fisher index to separate subaggregates because this index is not consistent in aggregation. In short, replication is the culprit.[6]

Notice that the GDP growth rate and component growth contributions in (8), and real shares in (10) are all independent of the base period $b$. This independence holds even when GDP in chained prices is computed from normally available data on GDP components in current prices and in constant prices for *s* and *t*,[7]

$$(p_{is}q_{is}\,,\,p_{it}q_{it}) \quad ; \quad (p_{ib}q_{is}\,,\,p_{ib}q_{it})\,. \tag{11}$$

The data in (11) suffice to compute the Fisher quantity index in (1) and price index in (7) and, therefore, suffice to convert GDP from constant to chained prices in (3). The computation requires the *cross-products* of prices and quantities and *ratios* of prices and of quantities from different periods. These can be obtained from (11) by,

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[5] BEA reports the non-additivity "residual" in standard GDP tables in its *Survey of Current Business*. This residual is the difference between overall GDP in chained prices and the sum of the GDP subaggregates. It could be positive, negative, or accidentally zero in periods different from the base period when it is zero.

[6] Moreover, BEA's replication is not fully consistent with the idea of GDP in chained prices because while the relative price effects between components of a subaggregate are taken into account those between components belonging to different subaggregates are ignored. This explains intuitively the non-additivity result.

[7] A GDP component in current prices such as $p_{it}q_{it}$ represents a subaggregate (e.g., "meat" comprising different kinds) in which case $p_{it}$ is "average price" (e.g., per pound of meat) and $q_{it}$ is "total quantity" (e.g., total meat in pounds). In this example, $p_{it}q_{it}$ is total meat in current prices and $p_{ib}q_{it}$ is total meat in constant prices. While the latter is in "money" units, the constant average price implies that the growth rate of $p_{ib}q_{it}$ equals the growth rate of the "physical" meat being represented. It is for this physical representation that components in constant prices are analytically useful. Moreover, it is arguable that components in constant prices are *necessary* because above the individual commodity level, they are the only economic data involving some aggregation that behave like physical quantities in terms of growth behavior. Hence, the data in current and in constant prices in (11) together are necessary and sufficient to compute GDP in chained prices.

$$\frac{p_{is}q_{is}}{p_{ib}q_{is}}=\frac{p_{is}}{p_{ib}} \quad ; \quad \frac{p_{is}}{p_{ib}}\times p_{ib}q_{it}=p_{is}q_{it} \ ; \quad \frac{p_{it}q_{it}}{p_{ib}q_{it}}=\frac{p_{it}}{p_{ib}} \quad ; \quad \frac{p_{it}}{p_{ib}}\times p_{ib}q_{is}=p_{it}q_{is} \ ; \quad (12)$$

$$\frac{p_{it}q_{it}}{p_{ib}q_{it}}\Big/\frac{p_{is}q_{is}}{p_{ib}q_{is}}=\frac{p_{it}}{p_{is}} \quad ; \quad \frac{p_{ib}q_{it}}{p_{ib}q_{is}}=\frac{q_{it}}{q_{is}} \,. \quad (13)$$

Note in (12) and (13) that base-period prices *cancel* out. This implies that the overall growth rate and growth contributions in (8), and real shares in (10) of GDP in chained prices are invariant with changes in the base period. Only the level of GDP in (3) will change because of the change in base-period GDP used as scalar.

## 3. GDP in constant prices as a special case of GDP in chained prices[8]

Relative prices are constant when all prices change at the same or constant periodic rate of $\pi \times 100$ percent. Since $t = s + 1$, it follows in this case that the prices in periods $s$, $t$, and $b$ are related by the proportionality equations,

$$p_{it}=(1+\pi)\,p_{is} \quad ; \quad p_{it}=(1+\pi)^{(t-b)}p_{ib} \quad ; \quad p_{is}=(1+\pi)^{(s-b)}p_{ib} \quad ; \quad \text{all } i\,. \quad (14)$$

Together, (1), (2), and (14) yield,

$$Q^F_{bt}=Q^L_{bt}=Q^P_{bt}=\frac{\sum_i^N p_{ib}q_{it}}{\sum_i^N p_{ib}q_{ib}} \quad ; \quad J_t=J_b\,Q^F_{bt}=\frac{\sum_i^N p_{ib}q_{it}}{\sum_i^N p_{ib}q_{ib}} \ \text{ since } \ J_b=1\,. \quad (15)$$

Therefore, by combining (15) with (3), GDP in chained and in constant prices are equal,

$$Y^F_t=\frac{\sum_i^N p_{ib}q_{it}}{\sum_i^N p_{ib}q_{ib}}\sum\nolimits_i^N p_{ib}q_{ib}=\sum_i^N p_{ib}q_{it} \quad ; \quad Y^F_s=\sum_i^N p_{ib}q_{is} \quad (16)$$

when relative prices are constant.

In turn, the growth rate of GDP from (16) is,

$$\frac{Y^F_t}{Y^F_s}-1=\frac{\sum_i^N p_{ib}q_{it}}{\sum_i^N p_{ib}q_{is}}-1=\sum\nolimits_i^N g^*_{it} \quad ; \quad g^*_{it}=\left(\frac{p_{ib}q_{is}}{\sum_i^N p_{ib}q_{is}}\right)\left(\frac{q_{it}}{q_{is}}-1\right) \quad (17)$$

where $g^*_{it}$ is a component's contribution to GDP growth in constant prices that differs, in general, from a component's contribution to GDP growth in chained prices given by $g^F_{it}$ in (8). However, $g^F_{it}$ and $g^*_{it}$ are equal,

$$g^F_{it}=w^F_{is}\left(\frac{q_{it}}{q_{is}}-1\right)=g^*_{it}=\left(\frac{p_{ib}q_{is}}{\sum_i^N p_{ib}q_{is}}\right)\left(\frac{q_{it}}{q_{is}}-1\right) \ \text{ since } \ w^F_{is}=\left(\frac{p_{ib}q_{is}}{\sum_i^N p_{ib}q_{is}}\right) \quad (18)$$

when relative prices are constant.

[8] Dumagan (2008) provides empirical illustrations using Philippine GDP data of the analytic results in sections 2 and 3. However, this earlier paper did not specifically recognize that GDP in constant prices is a special case of GDP in chained prices when relative prices are constant.

Moreover, recall the level contribution in chained prices given by $y_{it}^F$ in (9). By substituting $Y_s^F$ in (16) and $w_{is}^F$ in (18) into (9) and then simplifying, it can be verified that,

$$y_{it}^F = Y_s^F w_{is}^F \left(\frac{q_{it}}{q_{is}}\right) = p_{ib} q_{it} \quad ; \quad y_{is}^F = p_{ib} q_{is} \,. \tag{19}$$

Hence, when relative prices are constant, components in chained and in constant prices are equal so that their corresponding shares are also equal. That is,

$$\frac{y_{it}^F}{Y_t^F} = \frac{Y_s^F}{Y_t^F} w_{is}^F \left(\frac{q_{it}}{q_{is}}\right) = \frac{p_{ib} q_{it}}{\sum_i^N p_{ib} q_{it}} \quad ; \quad \sum_i^N \frac{y_{it}^F}{Y_t^F} = \sum_i^N \frac{p_{ib} q_{it}}{\sum_i^N p_{ib} q_{it}} = 1 \,. \tag{20}$$

However, having a fixed base period is problematic for GDP in constant prices when this base period is either *changed* or *maintained.*

If the base period is changed from *b* to *c*, the growth rate of GDP in constant prices will change if the prices in *b* and *c* are not proportional to each other. That is, from (14) and (16),

$$\frac{\sum_i^N p_{ib} q_{it}}{\sum_i^N p_{ib} q_{is}} - 1 \neq \frac{\sum_i^N p_{ic} q_{it}}{\sum_i^N p_{ic} q_{is}} - 1 \quad , \quad \text{if} \;\; p_{ic} \neq (1+\pi)^{(c-b)} p_{ib} \quad \text{for some } i \,. \tag{21}$$

Since price non-proportionality rules in reality, the change in the growth rate in (21) is inevitable. This change is problematic, however, because it implies a change in the volume of production when, in fact, there is none since only the base period was changed.

Moreover, the change in the growth rate above necessarily implies that the shares of components in constant prices will change. This follows because (21) yields,

$$\sum_i^N \left(\frac{p_{ib} q_{is}}{\sum_i^N p_{ib} q_{is}}\right) \frac{q_{it}}{q_{is}} \neq \sum_i^N \left(\frac{p_{ic} q_{is}}{\sum_i^N p_{ic} q_{is}}\right) \frac{q_{it}}{q_{is}} \,. \tag{22}$$

In turn, the above inequality implies that,

$$\frac{p_{ib} q_{is}}{\sum_i^N p_{ib} q_{is}} \neq \frac{p_{ic} q_{is}}{\sum_i^N p_{ic} q_{is}} \quad \text{for some } i \,. \tag{23}$$

This means that a change in the fixed base from *b* to *c* will change a component's share in the same period $s$ if price non-proportionality in (21) holds.

Moreover, maintaining a fixed base is also problematic for GDP in constant prices. The problem is substitution bias because the fixed base keeps prices constant. Consequently, too much weight is given to components for which relative prices have fallen but too little weight to components for which relative prices have risen. Hence, the growth contributions from (17) and real shares from (18) are overstated for components with falling relative prices but understated for those with rising relative prices.

However, converting GDP from constant to chained prices removes the fixed base by making period $s$—immediately preceding the current period $t$—the base. That is, the base period and relative prices are continuously updated, thereby, decreasing the weights of components with falling relative prices and increasing those of components with rising relative prices. Thus, GDP in chained prices removes the base-period dependence and substitution bias of GDP in constant prices.

## 5. Numerical example

There are two cases in the following example. One has constant relative prices where the prices of good 1, good 2, and good 3 increase 5 percent each period. The other has changing relative prices where the price of good 2 increases 6 percent but the price of good 3 decreases 6.5 percent each period. Everything else remains the same.

The third from the last panel of the following table shows that overall GDP and GDP components in constant prices remain the same whether relative prices are constant or changing because *only* the prices in the fixed base period (t = 0) are used. This constancy of prices leads to problematic results. For instance, the growth contribution formula in (3) implies that the use of constant base-period prices will *overstate* the growth contributions of good 3 whose price is decreasing (6.5 percent per period) at the same time that it will *understate* the growth contributions of goods 1 and 2 whose prices are increasing (5 and 6 percent per period). It also follows from the real share formula in (5) that beyond the base period the real shares of good 3 are overstated while those of goods 2 and 3 are understated. These problematic results are in addition to the base-year dependence of these growth contributions and real shares in constant prices. Hence, GDP in constant prices distorts the picture of the economy's growth and transformation.

In the table, the third and second panels from the last show that GDP in constant prices and GDP in chained prices are equal when relative prices are constant but they differ when relative prices change. Finally, the last panel shows that a component in constant prices may increase at the same time that this same component in chained prices may increase or decrease when its relative price is falling. Good 3, whose price is falling absolutely and relative to the prices of good 1 and good 2, illustrates this possibility where its level contribution rose slightly from 1,812.6 (t = 1) to 1,813.0 (t = 2) but fell to 1,807.4 (t = 3). This rise and fall of good 3 in chained prices may be explained by the fact that it has the highest quantity growth rate among the three goods, thus, compensating for the fall in its absolute and relative price by rising initially and then falling eventually as its quantity growth is swamped by its falling relative price.

Comparing contributions to level of GDP in constant and chained prices

| | t = 0 | t = 1 | t = 2 | t = 3 |
|---|---|---|---|---|
| Constant relative prices | | | | |
| Good 1: p1(t), increasing 5 % each period | 5.00000 | 5.25000 | 5.51250 | 5.78813 |
| Good 2: p2(t), increasing 5 % each period | 6.00000 | 6.30000 | 6.61500 | 6.94575 |
| Good 3: p3(t), increasing 5 % each period | 7.00000 | 7.35000 | 7.71750 | 8.10338 |
| p3(t)/p1(t) | 1.40000 | 1.40000 | 1.40000 | 1.40000 |
| p2(t)/p1(t) | 1.20000 | 1.20000 | 1.20000 | 1.20000 |
| p3(t)/p2t) | 1.16667 | 1.16667 | 1.16667 | 1.16667 |
| Changing relative prices | | | | |
| Good 1: p1(t), increasing 5 % each period | 5.00000 | 5.25000 | 5.51250 | 5.78813 |
| Good 2: p2(t), increasing 6 % each period | 6.00000 | 6.36000 | 6.74160 | 7.14610 |
| Good 3: p3(t), decreasing 6.5 % each period | 7.00000 | 6.54500 | 6.11958 | 5.72180 |
| p3(t)/p1(t) | 1.40000 | 1.24667 | 1.11013 | 0.98854 |
| p2(t)/p1(t) | 1.20000 | 1.21143 | 1.22297 | 1.23461 |
| p3(t)/p2t) | 1.16667 | 1.02909 | 0.90773 | 0.80069 |
| Quantities | | | | |
| Good 1: q1(t), increasing 5 % each period | 150 | 158 | 165 | 174 |
| Good 2: q2(t), increasing 6 % each period | 200 | 212 | 225 | 238 |
| Good 3: q3(t), increasing 7 % each period | 250 | 268 | 286 | 306 |
| Level of GDP in constant prices (t = 0, fixed base period) | 3,700.0 | 3,932.0 | 4,178.8 | 4,441.3 |
| (Constant or changing relative prices) | | | | |
| Contribution of good 1 = p1(0) x q1(t) | 750.0 | 787.5 | 826.9 | 868.2 |
| Contribution of good 2 = p2(0) x q2(t) | 1,200.0 | 1,272.0 | 1,348.3 | 1,429.2 |
| Contribution of good 3 = p3(0) x q3(t) | 1,750.0 | 1,872.5 | 2,003.6 | 2,143.8 |
| Level of GDP in chained prices (t = 0, base period) | 3,700.0 | 3,932.0 | 4,178.8 | 4,441.3 |
| (Constant relative prices) | | | | |
| Contribution of good 1 | 750.0 | 787.5 | 826.9 | 868.2 |
| Contribution of good 2 | 1,200.0 | 1,272.0 | 1,348.3 | 1,429.2 |
| Contribution of good 3 | 1,750.0 | 1,872.5 | 2,003.6 | 2,143.8 |
| Level of GDP in chained prices (t = 0, base period) | 3,700.0 | 3,931.2 | 4,175.6 | 4,433.7 |
| (Changing relative prices) | | | | |
| Contribution of good 1 | | 807.7 | 890.1 | 977.8 |
| Contribution of good 2 | | 1,310.9 | 1,472.4 | 1,648.4 |
| Contribution of good 3 | | 1,812.6 | 1,813.0 | 1,807.4 |

Note: The contributions of the goods to the level of GDP in chained prices at t = 0 cannot be computed for the case of changing relative prices because formula (17) for these contributions requires data in the preceding year which do not exist in the above example.

By taking into account changes in relative prices, GDP in chained prices portray a more realistic picture of the relative importance of GDP components in the economy's growth and transformation. This picture will not change with changes in the base year because base-year prices cancel out from the formulas for the GDP growth rate and for the contributions to growth and real shares of components in chained prices.

## 5. Conclusion

GDP in constant and in chained prices are alternative measures of *real* GDP. GDP in constant prices is real in the "physical" sense that it measures overall quantities *unadjusted* for relative price changes because it uses only the fixed base-period prices. In contrast, GDP in chained prices is real in the "economic" sense that it measures overall quantities *adjusted* for relative price changes that have real substitution effects.

This paper's framework for GDP in chained prices showed that GDP in constant and in chained prices are equal when relative prices are constant and that these two GDP measures differ only when relative prices change. This framework has a novel additive procedure for components in chained prices, counter to the prevailing view that these components are non-additive as exemplified by US GDP in chained prices (dollars). The procedure allows relative prices to change but when relative prices are constant, it yields components in chained prices that correspondingly equal components in constant prices and, thus, the procedure is consistent with the additivity of GDP in constant prices.

Changing or maintaining a fixed base period makes GDP in constant prices problematic. Changing the base period—when relative prices change between the old and new base—changes the GDP growth rate even with the same quantities, which is problematic because a growth rate change implies an overall quantity change. On the other hand, maintaining the base period keeps prices constant so that relative prices are also constant. This results in substitution bias by giving too much weight to GDP components with falling relative prices and too little weight to those with rising relative prices.

Finally, this paper showed that converting GDP from constant to chained prices makes the period—immediately preceding the current period—as the base. In effect, conversion removes the fixed base by continuous updating and allows relative prices to change and, thus, cures the problems of base-period dependence and substitution bias of GDP in constant prices.

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